Numeric Response Questions

Quadratic Equation

Q.1 If the solution of the equation, $4^x - 3^{x-12z} = 3^{x+1/2} - 2^{2x-1}, x \in R$ is $\frac{k}{2}$ then find k.

Q.2 Let $\mathrm{p}, \mathrm{q} \in \{1,2,3,4\}$. The number of equations of form $\mathrm{px}^2 + \mathrm{qx} + 1 = 0$ having real roots is k. Then find k.

Q.3 If $\mathrm{x}^2 + 3\mathrm{x} + 5 = 0$ and $\mathrm{ax}^2 + \mathrm{bx} + \mathrm{c} = 0$ have common root/roots and $\mathrm{a}, \mathrm{b}, \mathrm{c} \in \mathrm{N}$. Find the minimum value of $a + b + c$.

Q.4 Find the product of real roots of equation, $x^2 + 18x + 30 = 2\sqrt{x^2 + 18x + 45}$.

Q.5 If the roots of the equation $12x^2 - mx + 5 = 0$ are in the ratio $2:3$ and the value of m is $5\sqrt{k}$ then find k.

Q.6 Find the least integral value of k for which, $(k-2)x^2 + 8x + k + 4 > 0$ for all $x \in R$,

Q.7 If $\mathrm{x}^2 + \mathrm{px} + \mathrm{q} = 0$ is the quadratic equation whose roots are $\mathrm{a} - 2$ and $\mathrm{b} - 2$ where a and b are the roots of $x^2 - 3x + 1 = 0$, then find value of $p + q$.

Q.8 If sum of the non-real roots of $(x^2 + x - 2)(x^2 + x - 3) = 12$ is $100 - k$ then find k.

Q.9 If the product of roots of equation, $x^2 - 5kx + 2e^{4\ln k} - 1$ is 31 , then find sum of roots.

Q.10 If $x = 2 + 2^{25} + 2^{15}$, then find the value of $x^3 - 6x^2 + 6x$.

Q.11 The coefficient of x in the equation $x^2 + px + q = 0$ was taken as 17 in place of 13 and its roots were found to be -2 and -15 . Then find product of roots of the original equation.

Q.12 If one root of the equations $ax^2 + bx + c = 0$ and $bx^2 + cx + a = 0 (a, b, c \in R)$ is common, then find the value of $\left(\frac{a^3+b^3+c^3}{abc}\right)^3$.

Q.13 If α, β and γ are the roots of the equation $5x^3 - qx - 1 = 0, (q \in R)$ then find the value of $\frac{\alpha^2-3}{\beta\gamma} + \frac{\beta^2-3}{\gamma\alpha} + \frac{\gamma^2-3}{\alpha\beta}$

Q.14 If the difference of the roots of the equation $x^2 + ax + b = 0$ is equal to the difference of the roots of the equation $x^2 + bx + a = 0$, if $a \neq b$ then find $a + b + 4$

Q.15 Find the value of a for which the sum of the squares of the roots of the equation $x^2 - (a-2)x - a - 1 = 0$ assume the least value.

ANSWER KEY

1. 3.00	**2.** 7.00	**3.** 9.00	**4.** 20.00	**5.** 10.00	**6.** 5.00	**7.** 0.00
8. 101.00	**9.** 10.00	**10.** 2.00	**11.** 30.00	**12.** 27.00	**13.** 3.00	**14.** 0.00
15. 5.00						

Hints & Solutions

1. $4^x + 2^{2x-1} = 3^{x+1/2} + 3^{x-1/2}$

$2^{2x}\left(1+\frac{1}{2}\right) = 3^x\left(\sqrt{3}+\frac{1}{\sqrt{3}}\right)$

$2^{2x}\left(\frac{3}{2}\right) = 3^x\left(\frac{4}{\sqrt{3}}\right)$

$2^{2x-3} = 3^{x-3/2}$

$2^{2x-3} = (\sqrt{3})^{2x-3}$.

Hold if $2x - 3 = 0$

$x = \frac{3}{2}$

$\Rightarrow \frac{k}{2} = \frac{3}{2} \quad \Rightarrow k = 3$

2. Equation will have roots if $p^2 \geq 4q$

If $q = 1$, then $p^2 \geq 4q$ for $p = 2$, 3 and 4

$q = 3$, then $p^2 \geq 4q$ for $p = 4$

$q = 2$, then $p^2 \geq 4q$ for $p = 3$ and $p = 4$

$q = 4$, then $p^2 \geq 4q$ for $p = 4$

Total 7 possibilities.

3. The roots of $x^2 + 3x + 5 = 0$ are non real $\therefore$

Both equation are identical

$\therefore \frac{a}{1} = \frac{b}{3} = \frac{c}{5} = \lambda$

$a + b + c = 9\lambda$

$\therefore$ Min $(a + b + c) = 9$

4. $x^2 + 18x + 30 = 2\sqrt{x^2+18x+45}$

$\boxed{x^2+18x+30=t}$

$t = 2\sqrt{t+15}$

$t^2 - 4t - 60 = 0$

$t = 10, -6$

put $x^2 + 18x + 30 = 10$ put $x^2 + 18x + 30 = -6$

$\Rightarrow x^2 + 18x + 20 = 0$ not possible because LHS

so $P = 20$ is (–) at this value

5. $\frac{B^2}{AC} = \frac{(m+n)^2}{mn} \Rightarrow \frac{m^2}{60} = \frac{25}{6}$

$m^2 = 250$

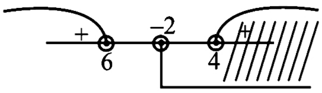
$m = 5\sqrt{10}$

6. $a > 0$ and $D > 0$

$k - 2 > 0$ and $64 - 4(k-2)(k+4) < 0$

$k > 2$ and $k^2 + 2k - 24 > 0$

$k > 2$ and $(k + 6)(k - 4) > 0$



$k \in (4, \infty)$

7. if, a, b are roots of Q.E

$x^2 - 3x + 1 = 0$ then Q.E whose root's $a - 2$ and $b - 2$ is $x \to x + 2$

$(x + 2)^2 - 3(x + 2) + 1 = 0$

$x^2 + x - 1 = 0$ compare with $x^2 + px + q = 0$ and get $p = 1$, $q = -1$.

8. Put $x^2 + x = y$, so that the equation (1) becomes

$(y - 2)(y - 3) = 12$

$\Rightarrow y = 6, -1$

When $y = 6$, we get $x^2 + x - 6 = 0$

or $x = -3, 2$

When $y = -1$, we get $x^2 + x + 1 = 0$

$\Rightarrow x = w, w^2$ and their sum is -1

9. $p = 2e^{4 \ln k} - 1 = 31$

$e^{\ln(k)^4} = 16$

$k = \pm 2$

As $k > 0$, so, $k = -2$

Now, sum $= 5k = 5 \times 2 = 10$

10. $x = 2 + 2^{2/3} + 2^{1/3}$ $x^3 - 6x^2 + 6x$

$x - 2 = (2^{2/3} + 2^{1/3})$ ...(1)

making whole cube

$(x-2)^3 = (2^{2/3} + 2^{1/3})^3$

$x^3 - 8 - 6x(x-2) = 4 + 2 + 3.[2^{2/3}.\ 2^{1/3}]$ $(2^{2/3} + 2^{1/3})$

$x^3 - 8 - 6x^2 + 12x = 6 + 6(x-2)$

$x^3 - 8 - 6x^2 + 12x = 6 + 6x - 12$

$x^3 - 6x^2 + 6x = 2$

11. $\boxed{p = 13}$ but when p = 17

Roots are –2 and – 15

$\therefore$ product is $\alpha.\beta = q$

$(-2)(-15) = q$

$\boxed{q = 30}$

$\therefore$ exact equation. $\Rightarrow x^2 + 13x + 30 = 0$

$\Rightarrow$ exact roots = – 3, –10

12. Applying condition for common root, we get

$a^3 + b^3 + c^3 = 3abc \Rightarrow \left(\frac{a^3+b^3+c^3}{abc}\right)^3 = 27$

13. We have $5x^3 - qx - 1 = 0$ → α, β, γ

Now, $\frac{\alpha^2-3}{\beta\gamma} + \frac{\beta^2-3}{\gamma\alpha} + \frac{\gamma^2-3}{\alpha\beta}$

$= \frac{\alpha^3 - 3\alpha + \beta^3 - 3\beta + \gamma^3 - 3\gamma}{\alpha\beta\gamma}$

$= \frac{\alpha^3+\beta^3+\gamma^3 - 3(\alpha+\beta+\gamma)}{\alpha\beta\gamma}$

$(\because \alpha + \beta + \gamma = 0 \Rightarrow \alpha^3 + \beta^3 + \gamma^3 = 3\alpha\beta\gamma)$

$= \frac{3\alpha\beta\gamma}{\alpha\beta\gamma} = 3.$

Aliter : $\alpha\beta\gamma = \frac{1}{5}$ and $\alpha + \beta + \gamma = 0$

$\therefore \left(\frac{\alpha^2-3}{\beta\gamma} + \frac{\beta^2-3}{\gamma\alpha} + \frac{\gamma^2-3}{\alpha\beta}\right)$

$= 5\alpha(\alpha^2 - 3) + 5\beta(\beta^2 - 3) + 5\gamma(\gamma^2 - 3)$

$= 5(\alpha^3 + \beta^3 + \gamma^3) - 15(\alpha + \beta + \gamma)$

$= 5(3\alpha\beta\gamma) = 15 \times \frac{1}{5} = 3$

[As $\alpha + \beta + \gamma = 0$]

14. $x^2 + ax + b = 0$ (let α, β),

$x^2 + bx + a = 0$ (γ, δ)

$\alpha - \beta = \gamma - \delta$

$\sqrt{(\alpha+\beta)^2 - 4\alpha\beta} = \sqrt{(\gamma+\delta)^2 - 4\gamma\delta}$ ...(1)

we have $\alpha + \beta = a$, $\alpha\beta = b$ and

$\gamma + \delta = -b$, $\gamma\delta = a$

then from eqn. (1)

$a^2 - 4b = b^2 - 4a$

$a^2 - b^2 + 4(a - b) = 0$

$(a - b)(a + b + 4) = 0$

$\Rightarrow a + b + 4 = 0$ (as $a \neq b$) Ans.

15. Let $y = \alpha^2 + \beta^2$

$y = (\alpha + \beta)^2 - 2\alpha\beta$

$= (a - 2)^2 + 2(a + 1)$

$= a^2 - 4a + 4 + 2a + 2$

$y = a^2 - 2a + 6$

$\therefore y' = 2a - 2 = 0$

$\therefore a = 1$

So $y = a^2 - 2a + 6$

$\therefore y_{min} = 1 - 2 + 6 = 5$